**5) <u>Using Integration to solve problems</u>**

A) Kinematics

- In Differentiation, you will be given an expression for distance and required to differentiate to find the velocity
- In Integration, you may be given an expression for the acceleration of an object and be asked to find the velocity, or you may be given the velocity and be asked to find the distance. In both cases you will need to integrate and find the constant of integration.

Worksheet

Example:

A skier begins down a slope and starts to accelerate. His acceleration can be modelled by the equation: $a = 0.48t^2 + 2$ for $0 < t < 10$

where $a$ is measured in $m/s^2$ and $t$ is the time in seconds from where the skier begins his descent. He was initially stationary. What is the velocity of the skier after 8 seconds?

$v = \int a(t)\ dt = \int 0.48t^2 + 2\ dt = 0.48\frac{t^3}{3} + 2t + c = 0.16t^3 + 2t + c$

So $v = 0.16t^3 + 2t + c$

Initially $t = 0$ and $v = 0$, therefore: $0 = 0 + 0 + c$ => $c = 0$.

The equation for velocity is therefore: $v = 0.16t^3 + 2t$

The velocity of the skier after 8 seconds is: $v(8) = 0.16(8)^3 + 2(8) = 97.92$ m/s

B) Differential Equations

Any equation involving derivatives or a derived function is called a **differential equation**. Differential equations are solved using integration. When a differential equation is solved, there are two types of solution:

- A **general solution** which will involve a constant.
- A **particular solution** where information is given, allowing the values of the constant to be calculated.

At Achieved level, the differential equation could simply be a derived question where you are expected to integrate and find the constant of integration, or you could be given an expression where you need to separate the variables. At Achieved level, the variables will be easy to separate.

Example 1: Find the solution to the differential equation $\frac{dy}{dx} = \cos(2x)$ if $y = 3$ when $x = \frac{\pi}{4}$.

If $\frac{dy}{dx} = \cos(2x)$, then $y = \int \cos(2x)dx$

$y = \frac{1}{2}\sin(2x) + c$

Sub ($y = 3, x = \frac{\pi}{4}$) into eqn: $3 = \frac{1}{2}\sin\left(2 \cdot \frac{\pi}{4}\right) + c$

$$3 = \frac{1}{2} \cdot 1 + c$$

$$c = 2.5$$

Therefore $y = \frac{1}{2}\sin(2x) + 2.5$

Example 2: Solve the differential equation $\frac{dy}{dx} = \frac{\sin(2x)}{4y}$.

If $\frac{dy}{dx} = \frac{\sin(2x)}{4y}$ then:

$4y\ dy = \sin(2x)\ dx$ *(separating the variables)*

$\int 4y\ \ dy = \int \sin(2x)\ dx$ *(integrating both sides)*

$\frac{4y^2}{2} = \frac{-1}{2}\cos(2x) + c$ *(constant of integration from LHS combined with RHS)*

$2y^2 = \frac{-1}{2}\cos(2x) + c$

$y^2 = \frac{-1}{4}\cos(2x) + \frac{c}{2}$

$y^2 = \frac{-1}{4}\cos(2x) + k$ *(renaming $\frac{c}{2}$ as $k$)*

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Delta Ex 24.2 pg 223 Q 1 - 8